

Activities with Graph Paper for Enhancing Learning of Mathematics

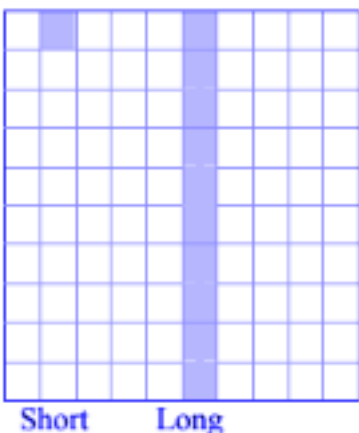
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Graph paper is a very versatile tool for learning concepts and procedures in mathematics but is not used much in elementary and middle schools. In this paper, we suggest some activities with graph paper that can be used to enhance understanding of concepts and operations on numbers, geometry, measurement, presenting data, drawing graphs of linear and quadratic functions. It provides another mode of representation that would be helpful for enhancing communication, problem solving, and further learning. It is almost free an important consideration for underdeveloped and developing countries. We have restricted activities to the syllabus for classes V to VIII in most countries.

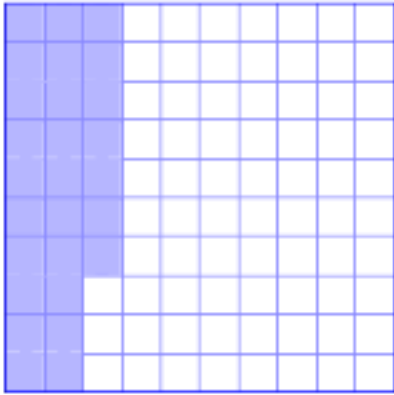
For all of these topics, the teacher is expected to give more examples and exercises to students.

Numbers

Let the **short**(a small square) shaded in the figure given below represent one unit of the area then the shaded **long** (a column of the square) represents a ten as it consists of ten squares and the **whole square** a hundred as it contains hundred squares.

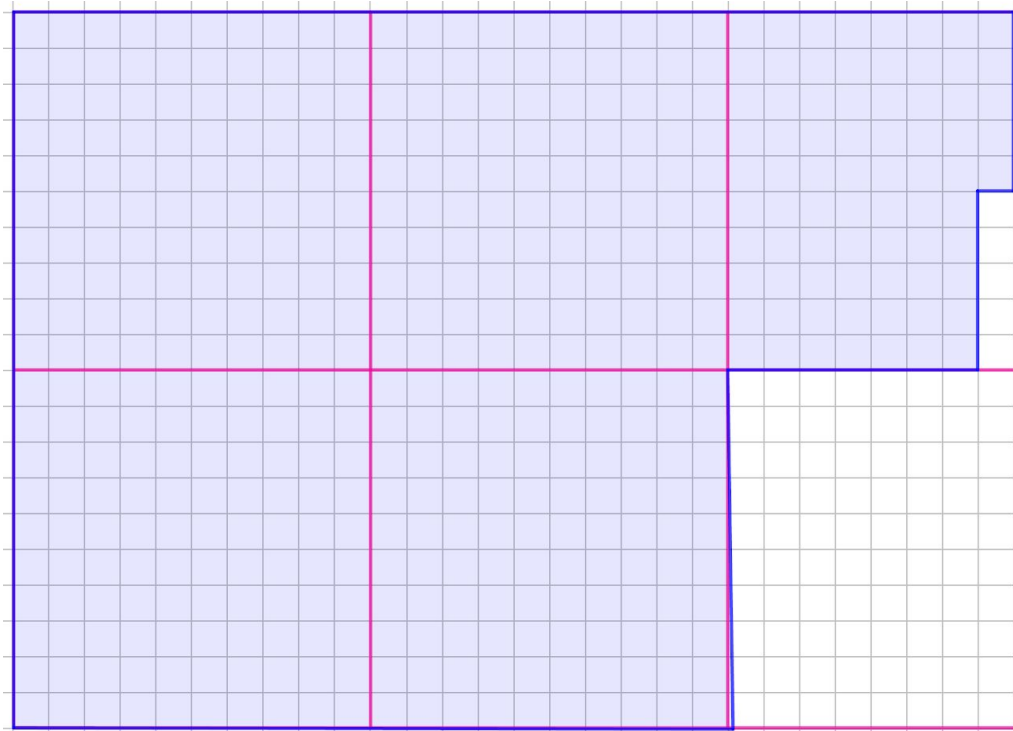


We can represent two-digit numbers by shading as many longs as tens and as many shorts as units in it. For example $27 = 2 \text{ Tens} + 7 \text{ Ones}$, we can represent it by shading 2 longs and 7 shorts as shown below:

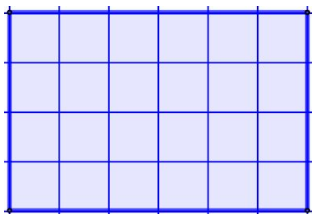


Similarly, we can represent a three-digit number by use of graph paper in which 10×10 squares are also marked.

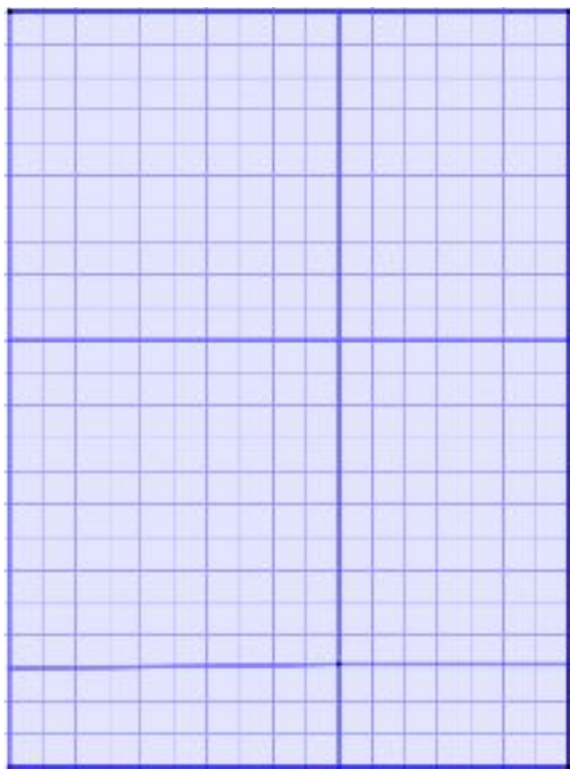
Then a big square represents a hundred, a long represents a ten and a short represents a unit. We can represent the number by shading as many 10×10 squares as hundreds, as many longs as tens, and as many shorts as units. Thus to represent a number say 475, we can shade four 10×10 squares, 7 longs, and 5 shorts as given below:



Multiplication Facts: Multiplication facts involve the multiplication of one-digit numbers. The area model for multiplication and division extends to fractions and decimals and should be used. Let the length of a small square represent one unit of length and its area one unit of area. To find 4×6 , mark and shade a rectangle with 4 rows and 6 columns on a graph paper and count the number of small squares in it. As it encloses 24 small squares, therefore $4 \times 6 = 24$.

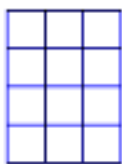


Multiplication of two-digit numbers: Mark a rectangle with as many columns as one number and as many rows as the other number. Then count the number of 10×10 squares, longs (include rows of 10 squares also), and shorts; and add them. For example to multiply 23×17 , Shade a 23×17 rectangle as given below and count the number of small squares in it by counting 10×10 squares, longs and shorts. The number of 10×10 squares or hundreds is 2, the number of tens is $14 + 3 = 17$, and the number of ones is 21, therefore the $23 \times 17 = 200 + 170 + 21 = 391$.



Properties of multiplication

Commutative law: Mark two rectangles 4×3 and 3×4 on the graph paper and count the number of squares. Both of them are 12. Therefore $4 \times 3 = 3 \times 4 = 12$.

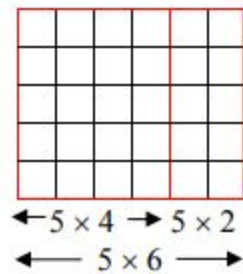


$$4 \times 3$$

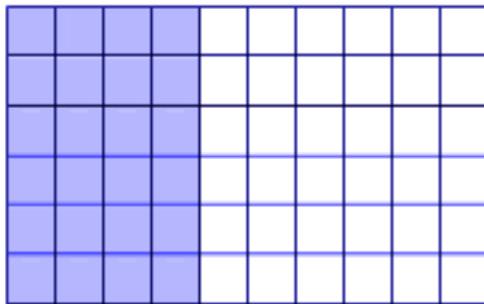


$$3 \times 4$$

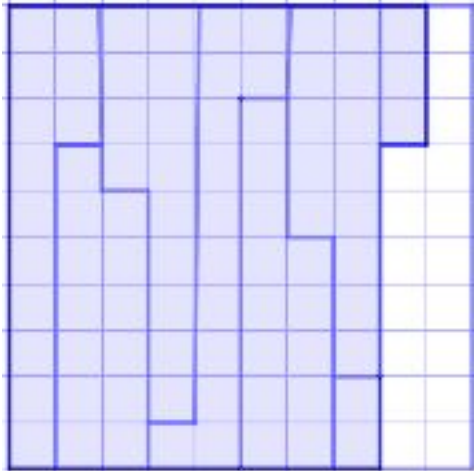
Distributive law The product of a number and sum of two numbers is equal to the sum of products of the number with each of the two numbers. For example $5 \times (4 + 2) = (5 \times 4) + (5 \times 2)$ as the figure given below shows



Division Facts: For example to find $24 \div 6$, Mark a rectangle using as many rows as 6 (the divisor) and go on shading columns having 6 rows till you have 24 units (the dividend). The number of columns gives the quotient. As it requires 4 columns to shade 24 units of an area using columns of 6 rows. Therefore $24 \div 6 = 4$.



Division of two-digit numbers by one or two-digit number: Use a graph paper in which 10×10 squares are also marked, shade an area equal to the **dividend**. Enclose as many small squares as the **divisor** by drawing lines around them, continue enclosing until less than 13 of the shaded squares are left. The number of enclosures gives the **quotient** and the leftover number of shaded shorts the **remainder**. For example, to divide 83 by 13, we enclose 13 squares in thicker lines and go on enclosing 13 squares. After we have enclosed 13 shorts 6 times, we have enclosed 78 shorts, and only 5 shorts are left, which are less than the divisor. The number of enclosures 6 gives the quotient and the 5 shorts that are left give the remainder. We can write it as $83 \div 13 = 6R5$, where R stands for the remainder.



Fractions

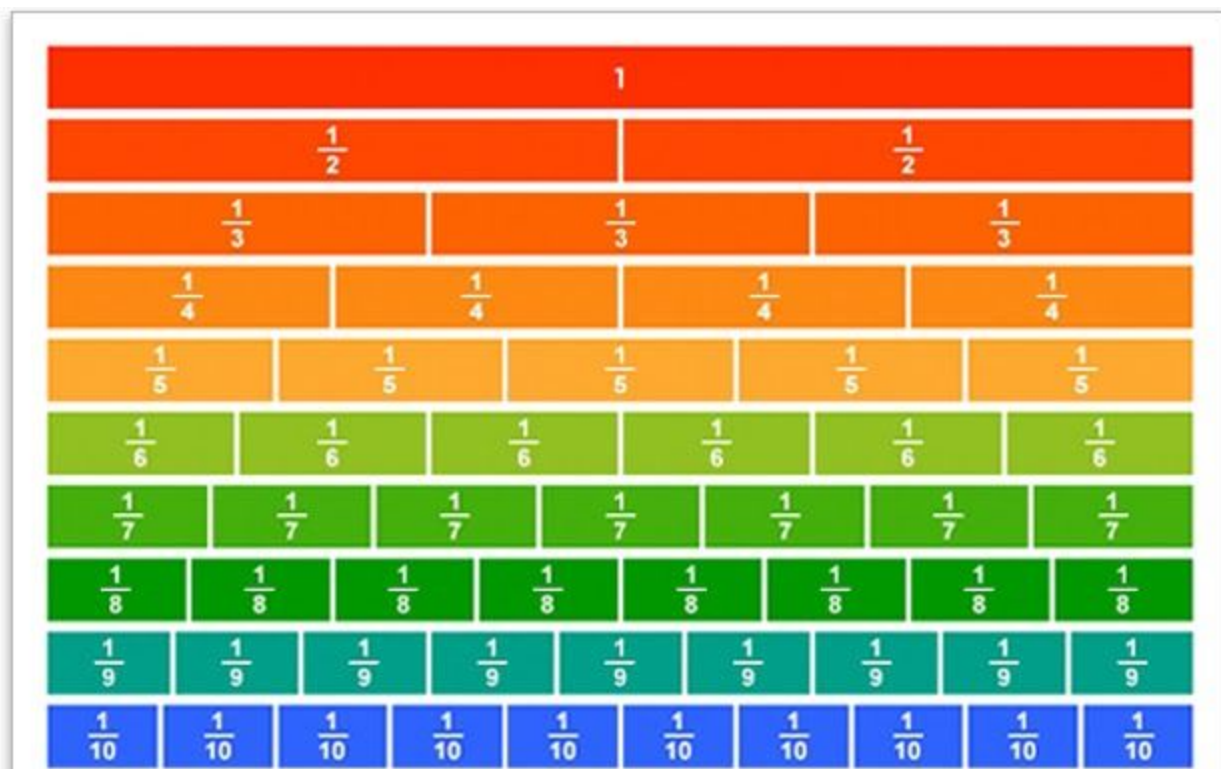
A fraction gives the number of parts under consideration from the number of parts in which a whole is divided. A fraction is written with a slash or (-) between the two numbers. For example, if a cake is divided into 8 equal parts and you have 3 of them, then you have $\frac{3}{8}$ (or $\frac{3}{8}$) of the cake.

Three is called the **numerator** of the fraction and 8 is called the **denominator** of the fraction.

Types of Fractions.

The fraction chart given below gives rectangles or bars having the same width and length. Taking the length of the bar as one unit, the bars are divided into 2, 3, 4,...parts.

Fraction Chart



What does one part of different colored bars represent?

What is common in these fractions? (They all have 1 as a numerator.) Such fractions are called **unit fractions**.

Compare the unit fractions. What do you notice? (The unit fractions with larger denominators are smaller.)

What does 1, 2, 3, ...parts in the 7th bar that is divided into 7 equal parts represent? ($\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, ...).

What is common in these fractions? (They all have the same denominator.) Such fractions are called **like fractions**.

Compare these fractions. What do you notice? (The fractions with larger numerators are larger.)

What does one part in the second bar represent? ($\frac{1}{2}$).

What do three parts in the sixth bar represent? ($\frac{3}{6}$).

Is the region covered by $\frac{1}{2}$ equal to that covered by $\frac{3}{6}$? (Yes.)

What does one part in the third bar represent? ($\frac{1}{3}$)

What do two parts in the third bar represent? ($\frac{2}{3}$) ?

What do four parts in the sixth bar represent? ($\frac{4}{6}$)

Is the region covered by $\frac{2}{3}$ the same as that covered by $\frac{4}{6}$? (Yes.)

The fractions that cover equal regions of equal figures are called **equivalent fractions**.

Are there any other equivalent fractions in the above figure?

Give rules for forming equivalent fractions.

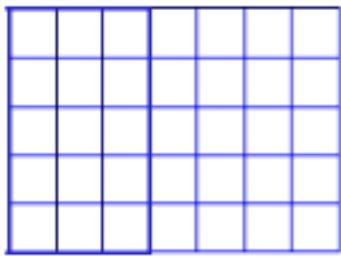
Fractions that have unlike denominators are called **unlike fractions**.

Comparison of unlike fractions

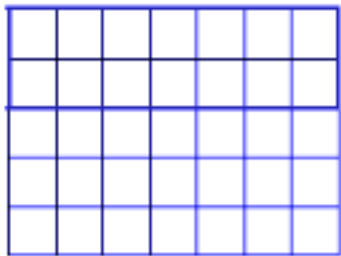
We can compare unlike fractions using the Fraction Chart. For example, if we look at the area covered by $\frac{3}{7}$ and $\frac{2}{5}$, the area covered by 3 squares in the unit bar divided into 7 parts is greater than the area covered by 2 squares in the unit bar divided into 5 parts. Therefore $\frac{3}{7} > \frac{2}{5}$.

If we do not have a fraction chart or have fractions that are not in the chart, then we may use the following method:

Mark two rectangles on a graph paper whose length and breadth are given by the denominators of the unlike fractions. Take the rectangle as a unit and mark or shade regions corresponding to fractions and count the number of small squares in each. The fraction that has a larger number of squares or larger area is larger. For example to compare $\frac{3}{7}$ and $\frac{2}{5}$, mark a rectangle whose length is 7 units and breadth 5 units. Taking this rectangle as a unit, mark $\frac{3}{7}$ th of the rectangle by drawing a border around 3 of 7 columns, and count the number of small squares in it which is 15. Mark another 7 × 5 rectangle and draw a border around 2 rows to show $\frac{2}{5}$ of the rectangle and count the number of small squares in it which is 14. Therefore $\frac{3}{7} > \frac{2}{5}$.



$$\rightarrow \frac{3}{7}$$



$$\rightarrow \frac{2}{5}$$

Improper Fractions and Mixed Numbers

A fraction in which the denominator is greater than the numerator is called a **proper fraction** e.g. $\frac{4}{6}$, $\frac{7}{8}$, $\frac{2}{9}$. A fraction in which the numerator is greater than the denominator is called an **improper fraction** e.g. $\frac{3}{2}$, $\frac{7}{4}$, $\frac{8}{3}$. These represent a number of equal parts into which many wholes are divided. For example, to find $\frac{4}{3}$ Mark 4 (numerator) rectangles with the same length and width 3 (the denominator) units and shade $\frac{1}{3}$ of the rectangles. That is one whole rectangle out of three and one-third of the remaining rectangle. The shaded part is composed of one whole rectangle and $\frac{1}{3}$ of a rectangle. We can express it as $1\frac{1}{3}$. Numbers that are a combination of a whole number and a fraction are called **mixed numbers**. Repeat with more examples.



Fractions in lowest terms: Fractions that do not have a common factor other than 1 are called fractions in lowest terms. A fraction whose numerators and denominators have common factors can be reduced to fractions with the lowest terms by dividing its numerator and denominator by the HCF of the numerator and denominator. For example in $\frac{4}{6}$ HCF of 4 and 6 is 2; we can reduce this fraction to its lowest terms, by dividing its numerator and denominator by 2. Now 2 and 3 do not have any common factor other than 1, therefore it is a fraction in lowest terms.

Addition of like fractions: We can add like fractions e.g. $\frac{1}{5}$ and $\frac{2}{5}$, by marking a 1×5 (one row and 5 columns) rectangle on a graph paper and dividing into 5 (denominator) equal parts. Then each part is equal to $\frac{1}{5}$. First shade one part of the rectangle that is $\frac{1}{5}$ and then shade 2 parts of the rectangle that is $\frac{2}{5}$. Now three parts that is $\frac{3}{5}$ of the rectangle is shaded.

Therefore $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$.

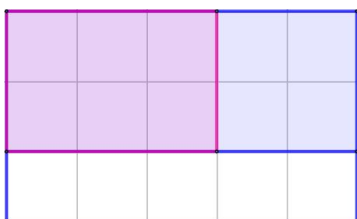
Note that the sum of two like fractions is a fraction whose numerator is the sum of numerators and the denominator is the same as denominators of like fractions.

Subtraction of like fractions: To subtract like fractions e.g. $\frac{2}{7} + \frac{4}{7}$, make a 1×7 rectangle on graph paper and first shade 4 parts (that is $\frac{4}{7}$) of the rectangle and then double-cross 2 parts (that is $\frac{2}{7}$) of the rectangle from that. Now only 2 parts (that is $\frac{2}{7}$) of the single crossed shaded rectangle is left. now Therefore $\frac{4}{7} - \frac{2}{7} = \frac{2}{7}$.

Note that the difference of two like fractions is a fraction whose numerator is the difference of numerators and denominator is the same as denominators of like fractions.

Addition and subtraction of unlike fractions: To add or subtract unlike fractions, find the LCM of their denominators. Change the fractions to equivalent fractions so that their denominator is the same as LCM. Now, these are like fractions and we can add and subtract them like those.

Multiplication of two fractions To multiply two fractions, say $\frac{2}{3}$ and $\frac{3}{5}$ mark a rectangle whose length is 5 linear units and breadth is 3 linear units (same as that of the denominators of two fractions). Taking the rectangle as a unit, shade $\frac{2}{3}$ rd of the area and then mark $\frac{3}{5}$ th of the area shaded earlier by a different colored border. The area within the red border represents 6 small squares out of a total of 15 or $\frac{6}{15}$ th of the area of the rectangle, therefore $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$.



Give the rule-The product of two fractions is a fraction whose numerator is the product of numerators of two fractions and whose denominator is the product of denominators of two fractions. Reduce it to the simplest form, if the numerator and denominator of the fraction have a common factor.

Division of fractions by a number: To divide a fraction by a whole number, say $\frac{1}{2}$ by 3, draw a 1×6 rectangle (6 here is the product of the denominator of the fraction and the number). Shade $\frac{1}{2}$ of that (that is 3 parts). Now divide it into three equal parts, each part will be equal to $\frac{1}{6}$. As there are three $\frac{1}{6}$ s in $\frac{1}{2}$, therefore $\frac{1}{2} \div 3 = \frac{1}{6}$.

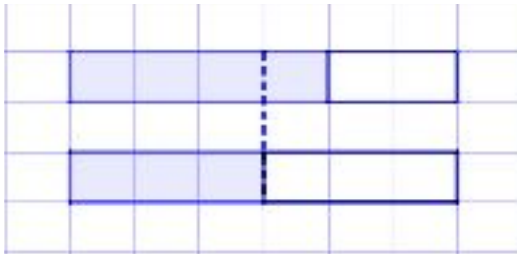


Division of fractions when the denominator of a fraction by which it is to be divided is a factor of the denominator of the fraction: To divide a fractions, say $\frac{1}{2}$ by $\frac{1}{10}$, draw a rectangle whose length is 10 linear units (same as that of the denominator of the smaller fraction) and breadth is 1 linear unit and take it as a unit. What does a square represent? ($\frac{1}{10}$). Shade $\frac{1}{2}$ the area and then find the number of $\frac{1}{10}$ s in it. As there are five $\frac{1}{10}$ s in it. Therefore $\frac{1}{2} \div \frac{1}{10} = 5$.



Division of fractions when the denominator of a fraction is not a factor of the denominator of the other fraction: To divide such fractions, say $\frac{2}{3}$ by $\frac{1}{2}$, draw two 1×6 bars. (6 is the product

of denominators), the second rectangle exactly below the first. Shade $\frac{2}{3}$ of the first bar and $\frac{1}{2}$ of the second bar. Then find out how many halves are there in $\frac{2}{3}$ as in the figure given below. You will notice there are $1\frac{1}{3}$ halves in it. Therefore $\frac{2}{3} \div \frac{1}{2} = 1\frac{1}{3}$.

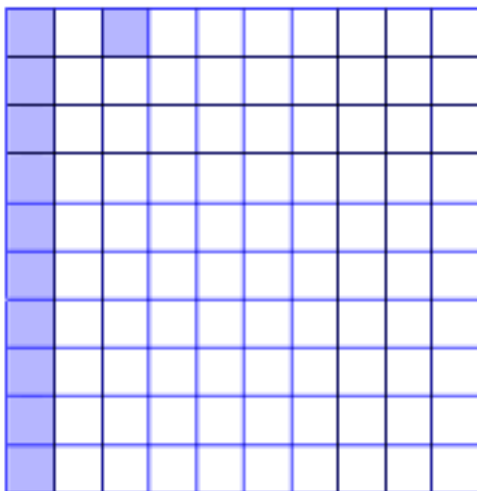


Representing irrational numbers that can be expressed as the sum of squares of two numbers on graph paper: To draw a line representing an irrational number say $\sqrt{17}$ find two numbers so that the sum of squares of these numbers is 17. As the sum of $4^2 + 1^2 = 17$. Draw two line segments AB and AC at right angles to each other whose lengths are 1 unit and 4 units along those lines. Join BC, then the length of the line segment $BC = \sqrt{1^2 + 4^2} = \sqrt{17}$ by Pythagoras Theorem.



Decimals

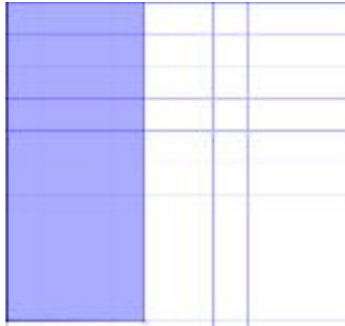
Decimals are special fractions that have 10, 100, 1000,... as denominators. Let a 10×10 square represents one unit, then the shaded long represents one-tenth($1/10$) and, the shaded short represents one-hundredth of the unit($1/100$).



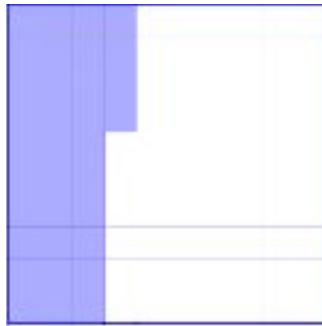
A number can be represented on graph paper by shading as many big squares as the number in the unit's place, as many longs as the number in tenth's place, and as many shorts as the number in hundredth's place.

The numbers involving decimal numbers are written by writing the whole number first, then a decimal point followed by tenths, and then hundredths from left to right.

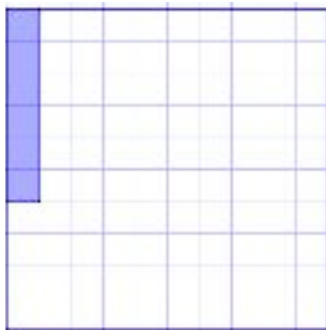
The examples given below show the numbers of shaded parts of the graphs taking a 10×10 square as a unit.



0.4



0.34



0.06



1.34

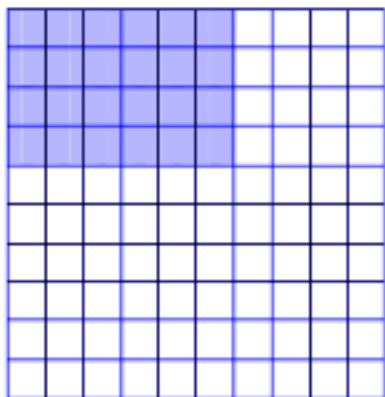
The numbers are read as zero point four, zero point three four(not thirty-four), zero point zero six, and one point three four.

Operations on Decimal numbers: In all the operations on numbers we would restrict to the graphical representation of operations on numbers involving at most two digits after the decimal. And take 10×10 square as a unit.

Addition: Shade the region corresponding to one of the numbers and then shade a region adjacent to that the region corresponding to the other number. The number represented by the total shaded region gives the sum of two numbers.

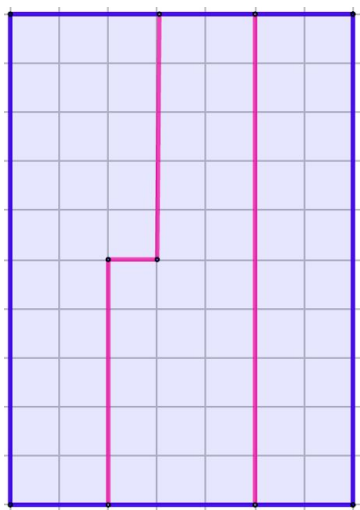
Subtraction: Shade the region corresponding to the minuend and cross out the region corresponding to the subtrahend. The number that represents the leftover shaded region gives the difference of two numbers.

Multiplication: The whole square has a length of 1; therefore the length of each short is one-tenth or 0.1, the length of 2 shorts is two-tenths or 0.2, the length of 3 squares is three-tenths or 0.3, and so on. To multiply 0.6 by 0.4, shade a rectangle whose length is 0.6 or 6 shorts and breadth is 0.4 or 4 shorts. As there are 24 shorts in the area or 24 hundredths, therefore $0.4 \times 0.6 = 0.24$.



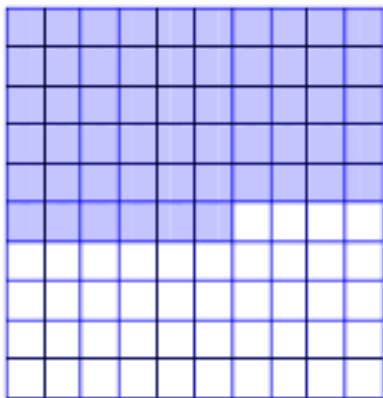
Similarly, we can extend it to multiplication of a whole number and a tenth and a number greater than one and a tenth.

Division: Shade the area in the graph paper that represents the dividend and then enclose the number of squares represented by the divisor. Continue doing that till the number left over is less than the divisor. The quotient is given by the number of enclosures and the remainder by the leftover squares. Express remainder by divisor as a decimal and add that to the quotient. For example, to divide 0.7 by 0.25, we first shade 7 longs and then enclose 0.25 that is 2 long and 5 shorts in it twice (enclosed by red borders). After we have done that only 2 longs or 20 shorts are left and $.20/.25 = 4/5 = 8/10 = 0.8$ adding that to the quotient we have 2.8, therefore $0.7 \div 0.25 = 2.8$

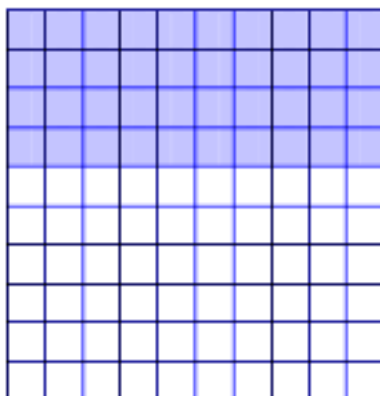


Percent means per centum or out of 100. We can represent a percent by using a 10×10 square and shading it. This square is divided into 100 equal parts, the number of shorts shaded parts give the percent of area shaded.

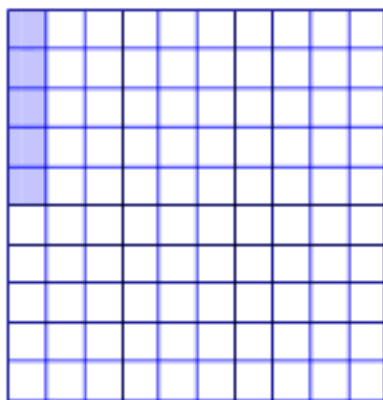
The percent of areas shaded in some figures are given below them.



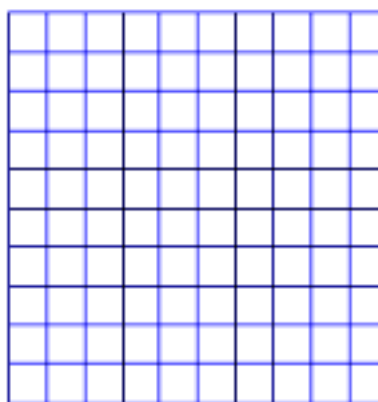
56%



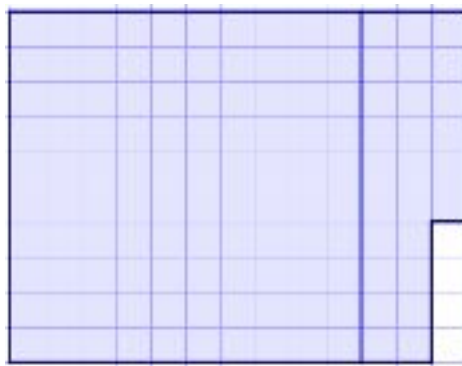
40%



5%



0%



126%

Geometry and Measurement

Use **triangular graph paper** given in Appendix 2 for all the activities given below:

Angle

Draw the following angles:

Draw acute, right, and obtuse angles.

Draw angles of 60° , 120° , 180° , 240° , and 300° .

Triangle

Draw an equilateral triangle.

If the side of the small triangle is taken as a linear unit of measurement, find the perimeters of a given equilateral triangle.

Draw equilateral triangles with specific perimeters e.g. 3, 9, and 15.

If the area of the small triangle is taken as a unit of area, find the area of a given equilateral triangle.

Draw equilateral triangles with specific areas e.g. 4, 9, and 25.

Polygons

Draw a rhombus.

If the side of the small triangle is taken as a linear unit of measurement, find the perimeter of the rhombus.

If the area of the small triangle is taken as a unit of area, find the area of a given rhombus.

Draw rhombuses with specific perimeters.

Draw rhombuses with specific areas.

Repeat all of the activities given for the rhombus for a parallelogram, trapezium, and hexagon, and for a square and a rectangle using square graph paper.

Use **Circular Graph Paper** given in Appendix 3 for the activities given below:

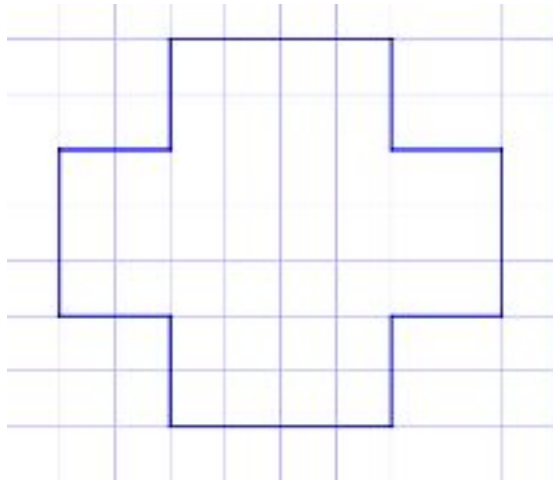
1. Draw a circle with a radius of 2 units.
2. Draw a sector of a circle with central angles equal to 20° , 60° , 90° , 120° , 130° , 240° , and 300° .
3. Find the area of a circle drawn on circular graph paper.
4. Draw a circle whose circumference is
 - (a) 2π .
 - (b) 4π .
5. Draw a circle whose area is
 - (a) π .
 - (b) 9π .
6. Find the area of a sector of a circle with a given central angle and radius.
7. Find the central angle of sectors whose area and radius are given.
8. Find the radius of a sector of a circle whose area and central angle are given.

Volume: We can make boxes of desired dimensions and find their volume and surface areas by using graph paper.

The four sides of the figure given below are folded to make an open box. Find its volume and surface area.

We can see that it will make a box whose length, width, and height will be 4 units, 3 units, and 2 units respectively. . Therefore its volume is $4 \times 3 \times 2 = 24 \text{ unit}^3$, where a small square represents the unit area

We can find the surface area by counting all the small squares in it. As there are 40 small squares in it, its surface area = 40 unit^2 .



Bar graphs

Bar graphs are used to present data given in a table in a graph which makes it easier to see the characteristics of the data.

The table given below gives data about the favorite TV program of students in a class.

Favorite TV Program of Students in a Class

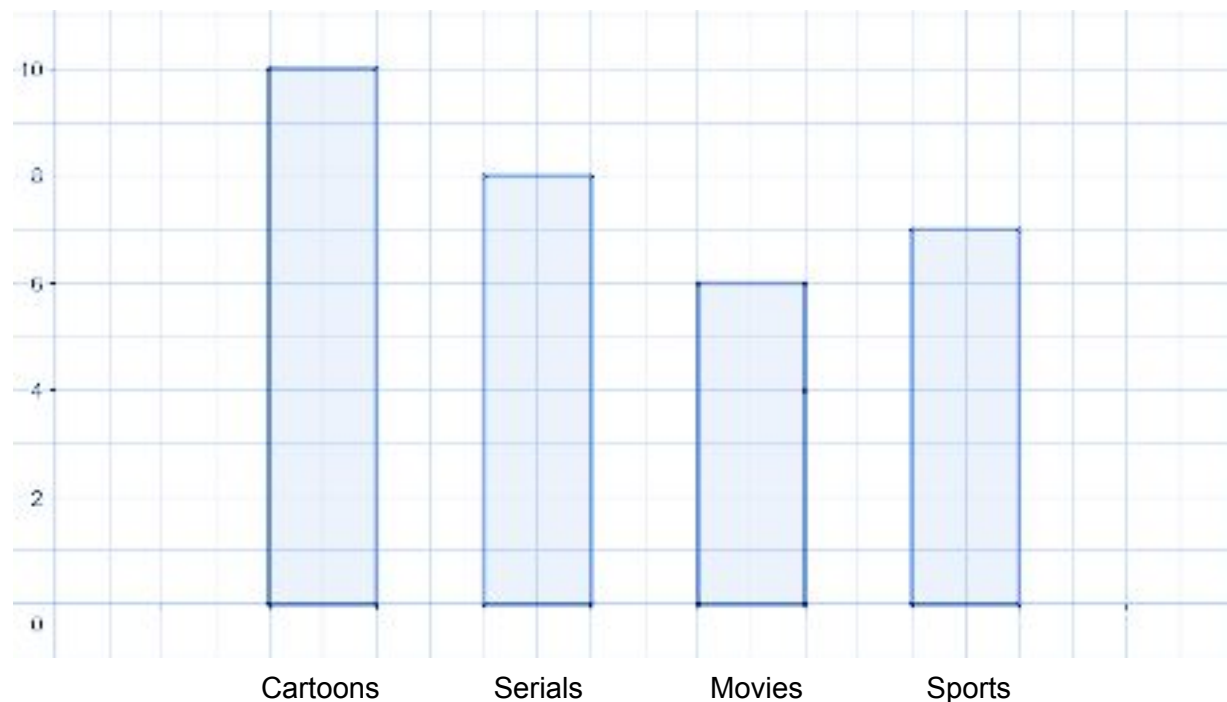
TV program	Number of students who said it was their favorite program
Cartoons	10
Serials	8
Movies	6
Sports	4

To draw a bar graph of the data given in the table

1. Draw two lines at right angles to each other on a graph paper.
2. Write the name of categories along the horizontal line (or vertical) and choose a scale for the length(or width) of the bar along the vertical (or horizontal) scale and write the numbers representing different lengths(or widths).

3. Erect bars of uniform width vertically (or horizontally) for each category with length equal to the number in a category using a square as a unit. The bars should have equal spacing between them, preferably the same as the length(or width) of the bars.
4. Label the horizontal(or vertical) axis by type of the program e.g. Cartoons, Sports,...and write the variable's label- Favorite TV Program below that.
5. Label the vertical(horizontal) axis-Number of students who reported it as their favorite program.
6. Write the title of the Bar Graph.

Favorite TV Programs of Students in a Class



This enables us to see more vividly

1. Number of children whose favorite program is a particular subcategory e.g. cartoons.
2. Which kind of program is most popular?
3. Which kind of program is least popular?
4. How many more children prefer to watch cricket matches than serials.

Histogram: It is similar to a Bar Graph, but a histogram is used for continuous data e.g. height, weight, marks in a test that is grouped into equal intervals. The height of each bar shows how many fall into each category.

The scores of 30 students in a Mathematics Test are summarized in the table given below:

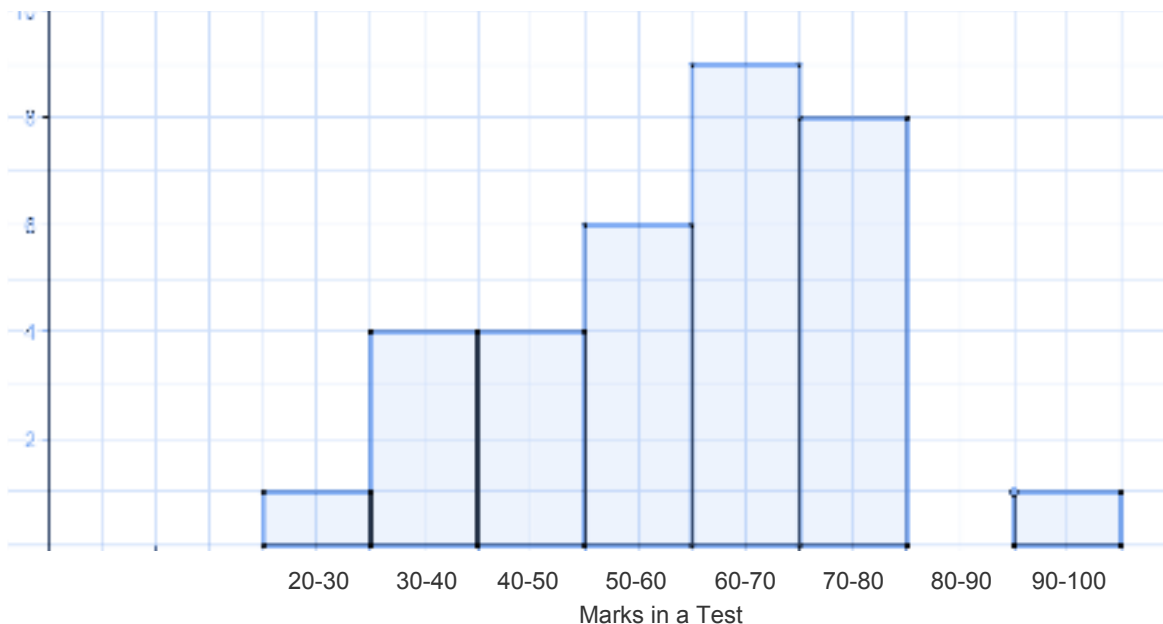
Marks of Students in a Test

Marks	Frequency
20-30	1
30-40	4
40-50	4
50-60	6
60-70	9
70-80	5
80-90	0
90-100	1

Although some numbers appear in both categories, the usual convention is the higher end of the interval is included in the next category.

The histogram of the data is given below:

Marks of Students on a Test



You can find from the histogram number of students who scored

(a) between 40 and 49 marks.

(b) above 70 marks.

(c) less than 40 marks.

Coordinate Plane.

A coordinate plane is a two-dimensional plane formed by the intersection of two lines perpendicular to each other in the middle of a graph paper.

The horizontal line is called the **x-axis** and the vertical line is called the **y-axis**.

The point of intersection of the x-axis and y-axis is called the **origin**.

The axes divide the plane into 4 parts called the **quadrants**.

The horizontal line toward the right of origin is the positive x-axis and the one toward the left is the negative x-axis.

The vertical line above the origin is a positive y-axis and the vertical line below the origin is a negative y-axis.

The **x-coordinate** is its perpendicular distance from the y-axis measured along the x-axis.

The **y-coordinate** is its perpendicular distance from the x-axis measured along the y-axis.

To plot an ordered pair (x,y) e.g. $(2,3)$ in the coordinate plane, move 2 units to the right as the coordinate is positive and then 3 units up on the y-axis, this point has the x-coordinate of 2 and y-coordinate of 3, it is written as $(2,3)$. (The x-coordinate is always listed first in the ordered pair)

Scatter Plot: A scatter plot has points that show the relationship between two sets of data. It can be made in a coordinate plane by plotting points (x,y) , where x and y are values of two variables for an observation.

Correlation

We can then check if a line close to most of the observations can be drawn.

The closeness of the points to a line gives the extent to which the variables are related and is called **Correlation**.

We can draw a line that seems to fit the data best. (That is close to most of the observations, some will be above and some below it.)

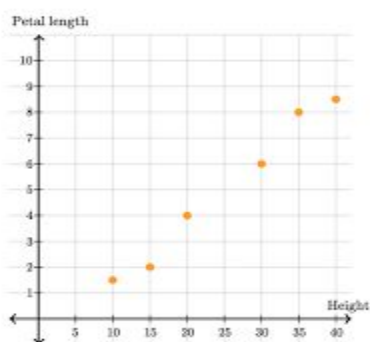
It is high(or low) if most of the points are very(or not) close to the line.

The correlation is positive(or negative) if one variable increases when the other variable increases(or decreases).

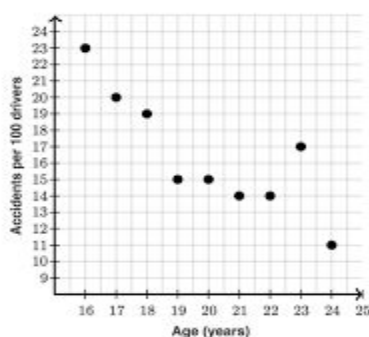
It is always between -1 and 1. It is 1 if the correlation is positive and all points are on a line and is -1 if the correlation is negative and all points are on a line. It is 0 if only a line parallel to x-axis seems to be the best fit.

You will learn later how to find the equation of the line of best fit and the formula of the value of the correlation coefficient.

Scatter Plots of Some Data



Positive High Correlation



Negative High Correlation

Algebra

Graphs of linear equations

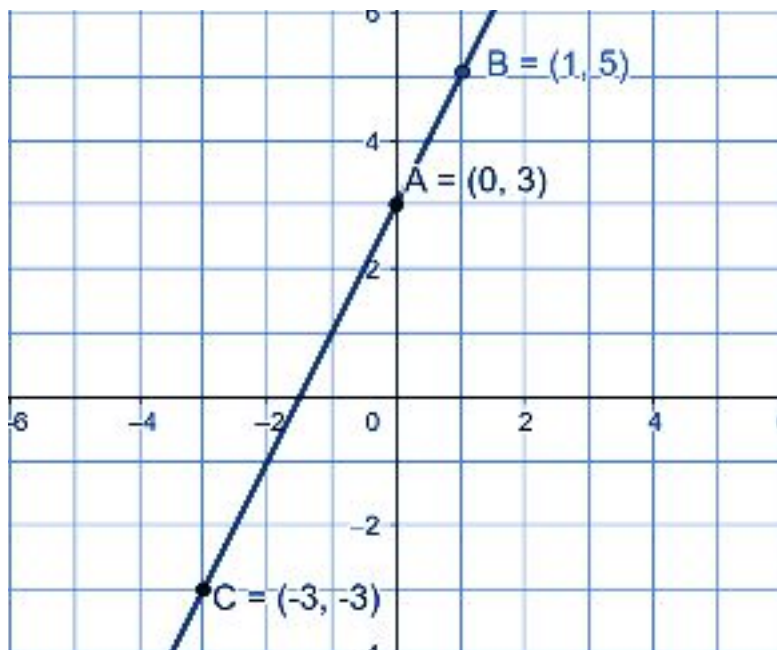
A linear equation in two variables is of the form $y = mx + b$ and is called **the Slope-Intercept Form** or $ax + by = c$, called **the Standard Form**, where m , a , b , c are any rational numbers. (Some define Standard Form as $ax + by + c = 0$.)

If the equation is given in the form $y = 2x + 3$, then to draw its graph

1. Find any three points whose coordinates satisfy the equation. The points $(1,5)$, $(2,7)$ and $(-1,2)$ satisfy the equation.
2. Arrange them in a table giving the corresponding values of x and y .

x	y
1	5
0	3
-3	-3

3. Plot the points $(1,5)$, $(0,3)$ and $(-3,-3)$ in the coordinate plane.
4. Draw a line passing through the three points.



All points on the line are solutions to the equation and all solutions are points on the line.

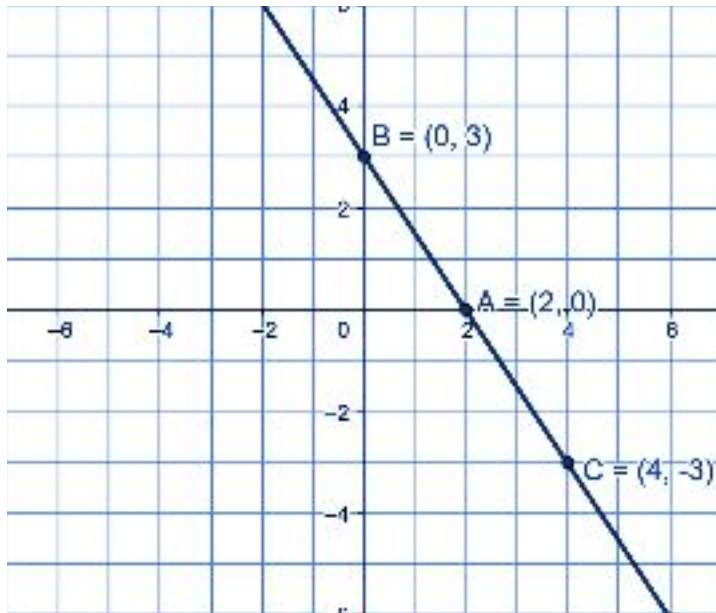
Note: Although two points determine a line, it is desirable to plot three points. Because if you plot one of the points wrong, a line can still be drawn through two points, but it will not be a line on which all the solutions to the equation lie.

If the equation of the line is in **Standard Form** viz. $ax + by = c$.

Then find the **x-intercept**(the value of x , when $y = 0$) and **y-intercept**(the value of y , when $x = 0$) and another point that satisfies the equation. Plot these points and draw a line through them.

For example to draw the graph of $2y + 3x = 6$,

1. Find the x-intercept i.e. value of x if $y = 0$, that is $x = 2$ and thus $(2,0)$ is a point on the line.
2. Find the y-intercept i.e. the value of y if $x = 0$, that is $y = 3$, thus $(0,3)$ is a point on the line.
3. Find another point that satisfies the equation. The point $(4,-3)$ satisfies the equation.
4. Plot the points $(2,0)$, $(0,3)$ and $(4,-3)$.
5. Draw a line passing through these points that will give the graph of the equation.



If the equation has only one variable e.g. $x = 4$ or $y = -3$, then the line is parallel to the y -axis or x -axis.

For $x = 4$, the graph is a line parallel to the y -axis at $x = 4$. All points on the line will have an x -coordinate equal to 4

For $y = -3$, the graph is a line parallel to x -axis at $y = -3$. All points on the line will have a y -coordinate equal to $y = -3$.

Graphs of inequalities in two variables

A point, for example, 5 separates the number line into two regions, in one part the numbers are greater than 5, and in the other part, numbers are smaller than 5.

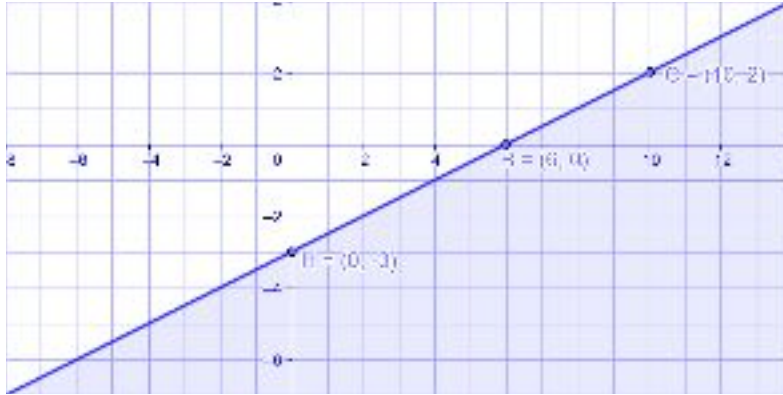
Similarly, a line e.g. $y = 2x - 1$, separates the points into two regions.

One region are points for which $y > 2x + 1$, and the other region has points for which $y < 2x + 1$.

$Y = 2x + 1$ is called a **Boundary Line**. In inequalities in two variables, the boundary line is shown with a **dashed** or **solid** line depending on whether the points on the line are to be excluded or included in the region analogous to a hollow or solid small circle on the number line.

We draw the graph of $y = 2x + 1$ as explained earlier and use a point in the region above or below the line to check if the inequality holds. If it holds, we shade that region, otherwise the opposite region and make the line solid as here the points where $y = 2x + 1$ hold are to be included in the region.

The shaded region in the graph given below is the solution to the inequality $y \leq 2x + 1$. Any point in the region satisfies the inequality and a point that satisfies the inequality is in the region.



Finding a solution of simultaneous equations in two variables by graphical method.

To find the solution of two simultaneous equations, draw graphs of both equations.

If the lines (or when produced) intersect, the coordinates of the point of intersection is the solution to the equation. It satisfies both equations.

If the lines do not intersect, then they are parallel and do not have a solution. This will be so if the lines in slope-intercept form differ only in the constant (y-intercept) and c in the standard form sometime after simplification of one of the equations.

If both the lines coincide, then they have an infinite number of solutions. This happens if one equation is the same or can be obtained by multiplying by a constant the other equation.

Graph of a quadratic equation

A quadratic equation is of the form $y = ax^2 + bx + c$, and it is a curve called **Parabola**.

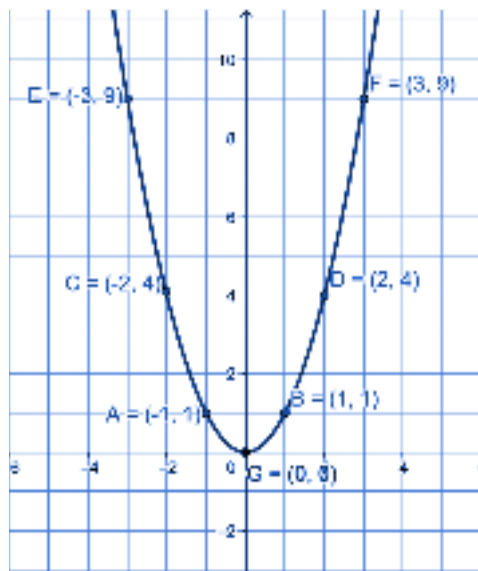
Let us consider the simplest case $y = x^2$. Find the corresponding values of y for some values of x e.g. 0, 1, 2, 3, -1, -2, -3.

1. Arrange them in a table giving the corresponding values of x and y .

x	y
0	0
1	1
2	4
3	9
-1	1
-2	4
-3	9

2. Plot these points in the coordinate plane.

3. Draw a smooth curve passing through these points.



The curve shows that:

1. If a point (x, y) satisfies it, then $(x, -y)$ will also satisfy it. It is symmetric about a line(y-axis here). The line is called **Axis of Symmetry** of the parabola
2. Its lowest point in G is called the **Vertex** of the parabola and is a **minima**(The minimum value y can have).
3. The curve extends to infinity.

The graph of any quadratic equation of the form $y = ax^2 + bx + c$ will have this shape.

It will open upwards if a is positive and downwards if a is negative.

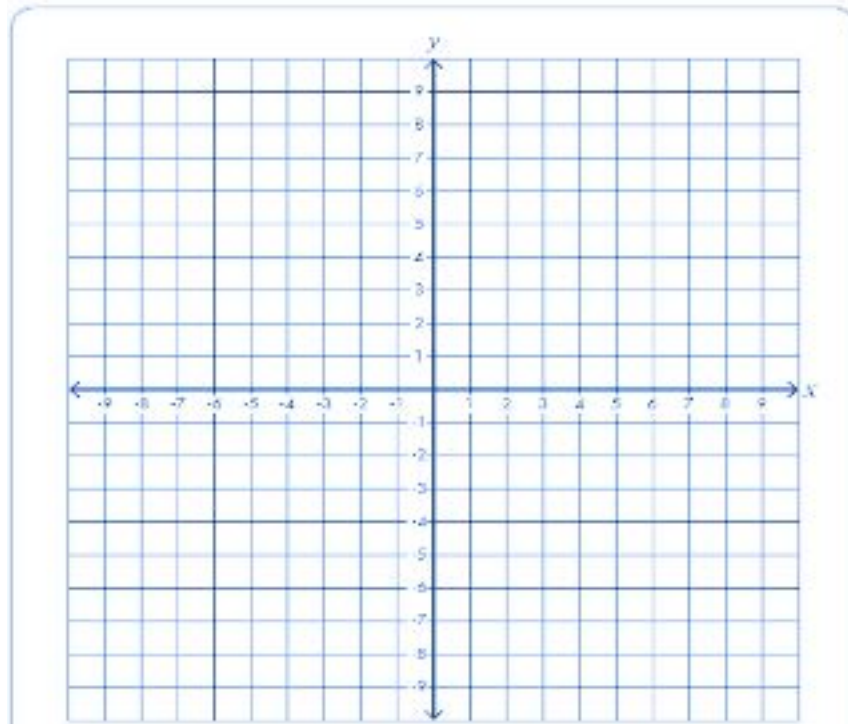
Its vertex will be at a point at the origin if $c = 0$, above the y -axis if $c > 0$, and below the x -axis if $c < 0$.

The vertex will be a **minima**(the minimum value y can have) if it opens upwards and a **maxima**(the maximum value y can have), if it opens downwards.

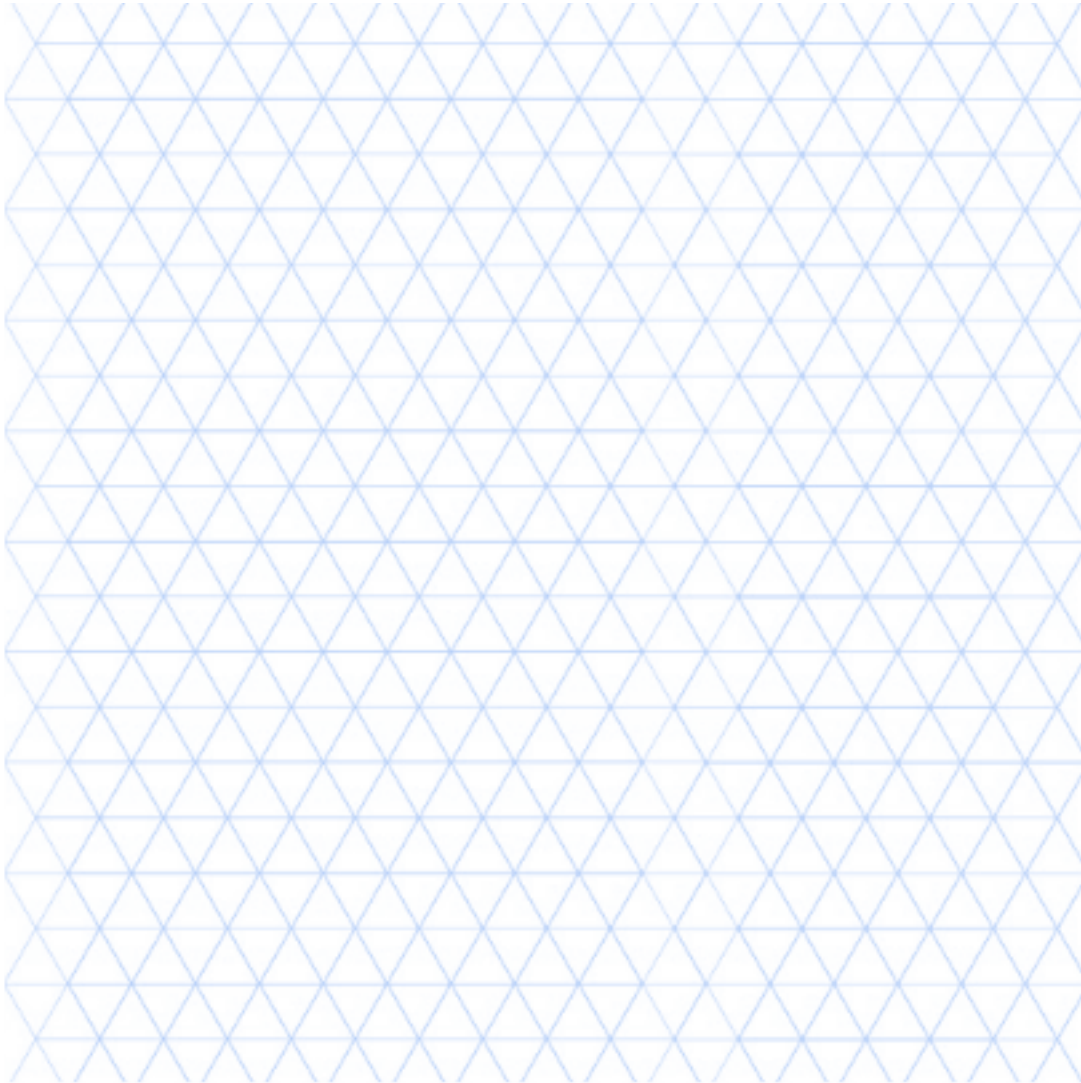
If the vertex is below the x -axis it will intersect it at two points, and will not intersect it if it is above the x -axis. The points where $y = 0$ can be found by finding the solution to $ax^2 + bx + c = 0$ and are called **x-intercepts**.

It will be wider if a is small and narrower if a is large.

Appendix 1



Appendix 2



Appendix 3

